1 to 9. But 1 multiplied by 10 produces 10, and 9 multiplied by 10 produces 90. And so on for all the intermediate units.

X. *A Recommendation of* Hadley's *Quadrant for surveying, especially the surveying of Harbours, together with a particular Application of it in some Cases of Pilotage. By the Rev.* John Michell, *B. D. F. R. S.*

Read Feb. 14, 1765.

THE use of Hadley's quadrant, as an instrument to take altitudes at sea, is already so well established, that it wants no farther recommendation ; but there are several other purposes, to which it may be applied, with great advantage, which, though obvious enough, seem yet to be hardly sufficiently attended to. There is no instrument so well adapted to many kinds of surveying, either for exactness or conveniency, and particularly the last; but the surveying of harbours, or such sands, as lie within sight of land, may oftentimes be performed by it, not only with vastly more ease, but also with a much greater degree of precision, than can be hoped for by any other means, as it is the only instrument in use, in which neither the exactness of the observations, nor the ease with which they may be taken, are sensibly affected by the motion of a vessel : and hence a single observer, in a boat, may generally determine the situation of any place

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he pleafes, with a fufficient degree of accuracy, if, with this inftrument, he takes the angles fubtended by two or three pairs of objects properly chofen upon the fhores round about him; but it will be ftill better to have two obfervers, one of whom, being in a boat, muft, at the time he takes the angle fubtended by fome two objects upon the fhore, make a fignal to the other obferver, who, being placed at one of the objects, as a ftation, muft at the fame time obferve the angle fubtended by the boat and the other object. By this means, two angles in a plain triangle being given, together with the diftance between the two objects, as a bafe, the whole triangle, and the fituation of every part of it, will be given likewife. By fuch obfervations, as thefe, provided the boat be at reft during the time of making them, and they be made carefully, with good quadrants, though without the affiftance of telefcopic fights, the fituation of places may be eafily determined to twenty or thirty feet upon every three or four miles.

Befides the ufe of Hadley's quadrant in furveying, it may upon fome occafions be very advantageoufly employed in piloting fhips into harbours, the great readinefs, with which it may be ufed, making it a very convenient inftrument for this purpofe: but that this may be done to the greateft advantage, it will be neceffary to have a proper provifion made for it upon the charts, by expreffing upon them the angles fubtended by given objects, by means of which, together with the bearings, a fhip may be enabled to know her fituation with great exactnefs. The well-known property of the circle, that angles in the fame fegment are equal to each other, may be often-

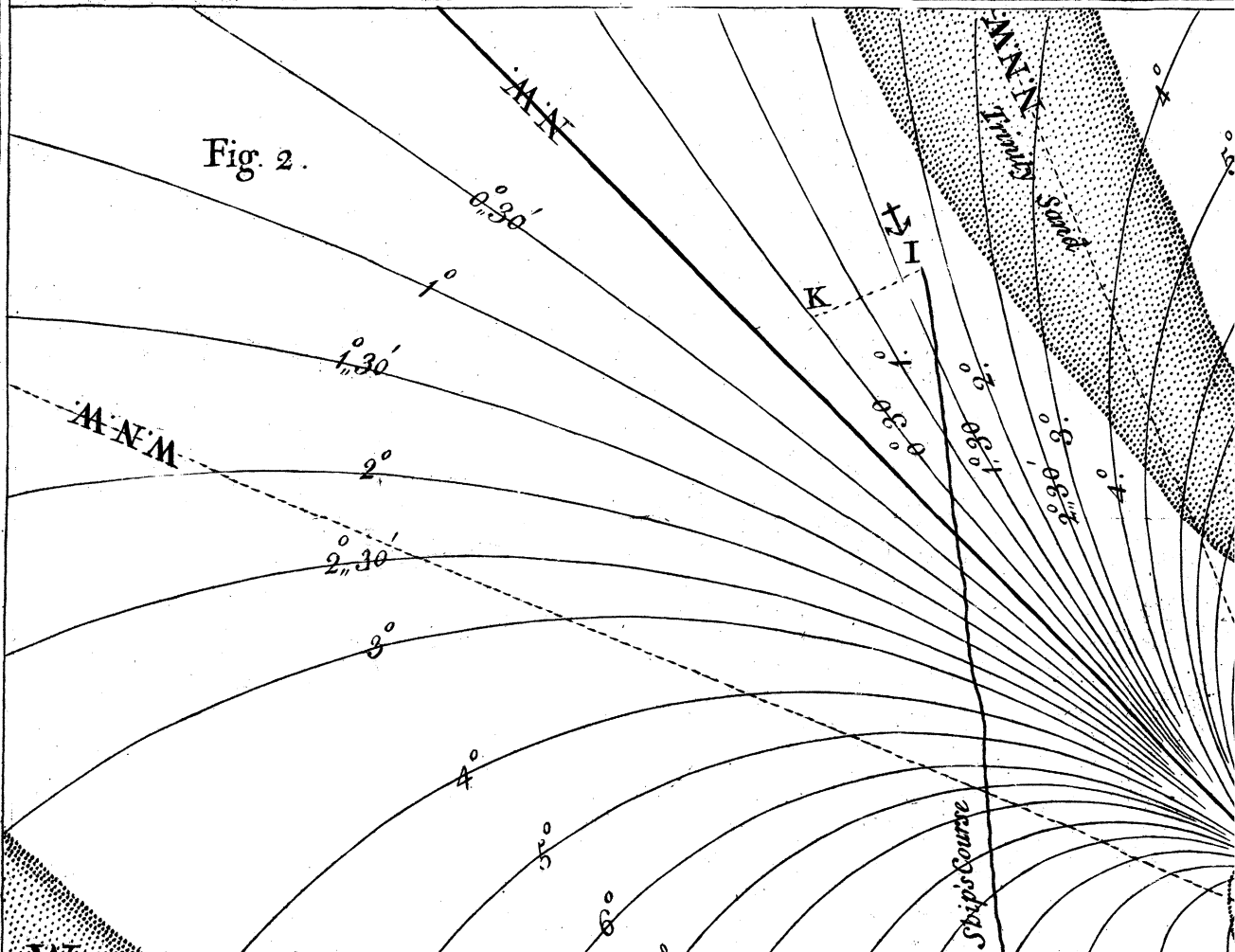<div align="right">times</div>

times very conveniently applied upon this occasion ; for if, through any two given objects, we describe several segments of circles, in which those objects shall subtend the angles of * 120°, 90°, 80°, 70°, &c. respectively, we shall then know immediately, upon finding the two objects subtend any one of these angles, that we are situated somewhere in the circumference of the corresponding segment ; and, the bearing also from one of the objects being known, our precise situation will be determined with great accuracy.
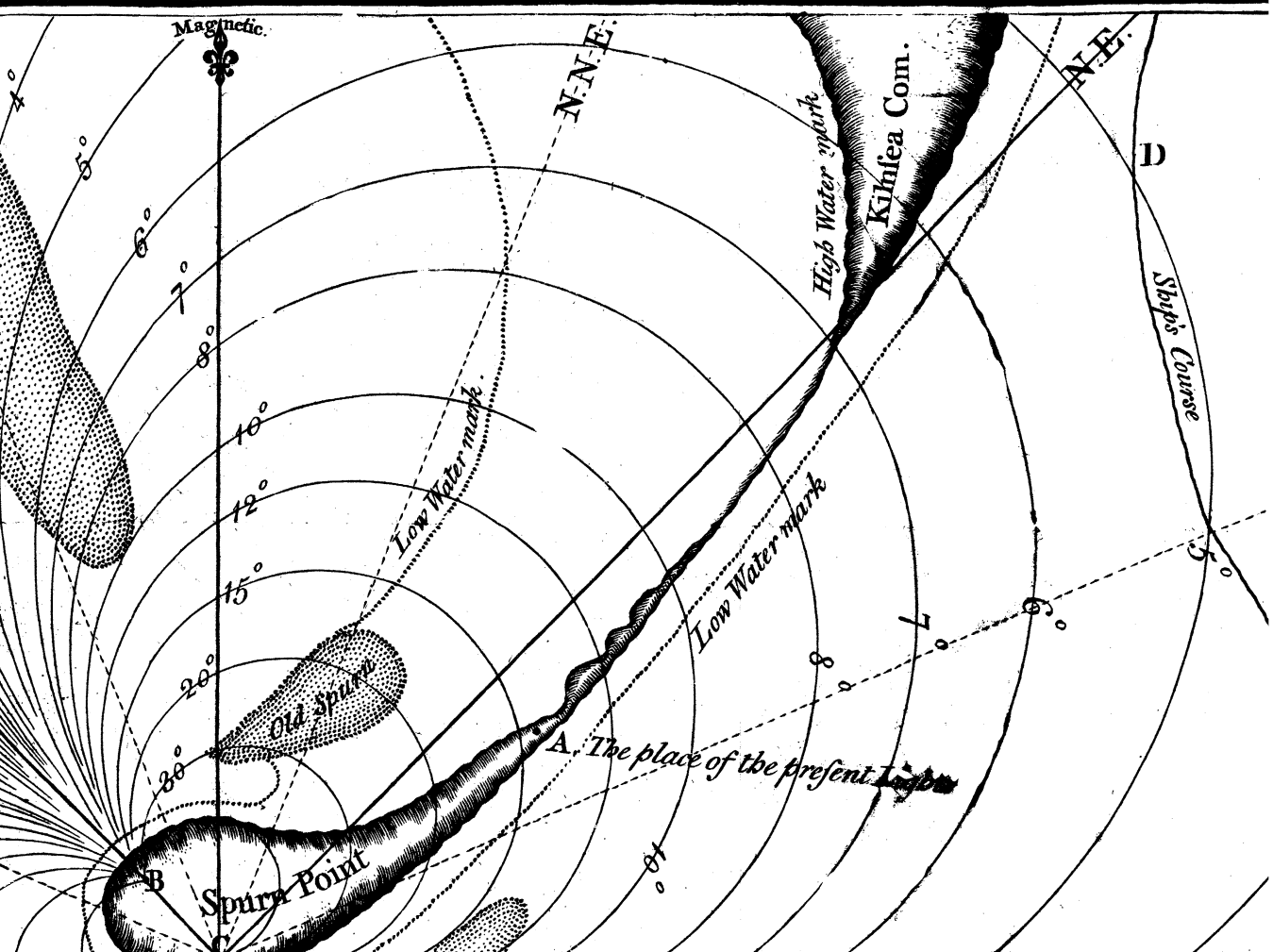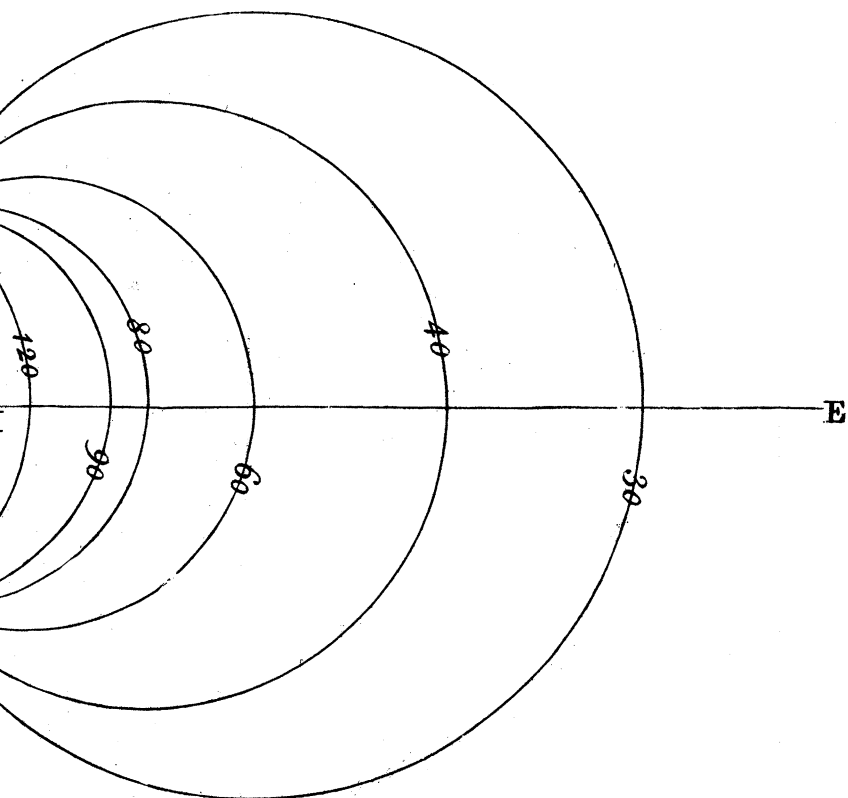
The manner of describing these segments, through two given points, may be performed in the following manner. Let B and C [TAB. II. fig. 1.] be the given points : then, joining these two points, bisect the line B C in A, and draw the indefinite right line DE, perpendicular to B C, through the point of bisection. Upon BC, at the point C constitute the angles D C B, F C B, G C B, H C B, &c. respectively equal to the difference between the angles, which correspond to the several intended segments and 90°; and on the opposite or same side of the line C B with those segments, accordingly as they exceed or fall short of 90°. Then will the points D, F, G, H, &c. where the lines C D, C F, C G, C H, &c. intersect the line D E, be the centres of
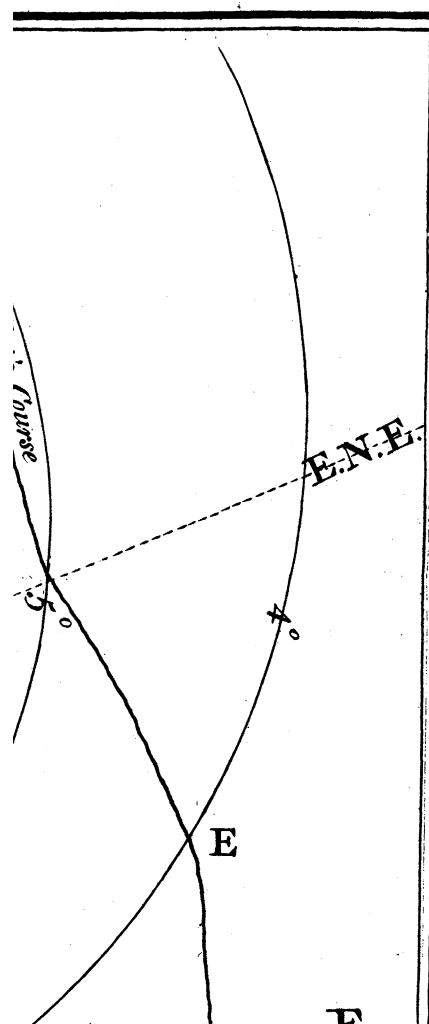
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* The number and frequency of these segments, as well as the magnitude of their respective angles, must be determined, according to the particular circumstances of the occasion, upon which they are applied : I have mentioned no greater angle, than 120°, as there are few cases, in which this will not be sufficient ; and indeed it is the greatest, that Hadley's quadrant, the only instrument fit for this purpose, will easily admit, according to the present construction of it.

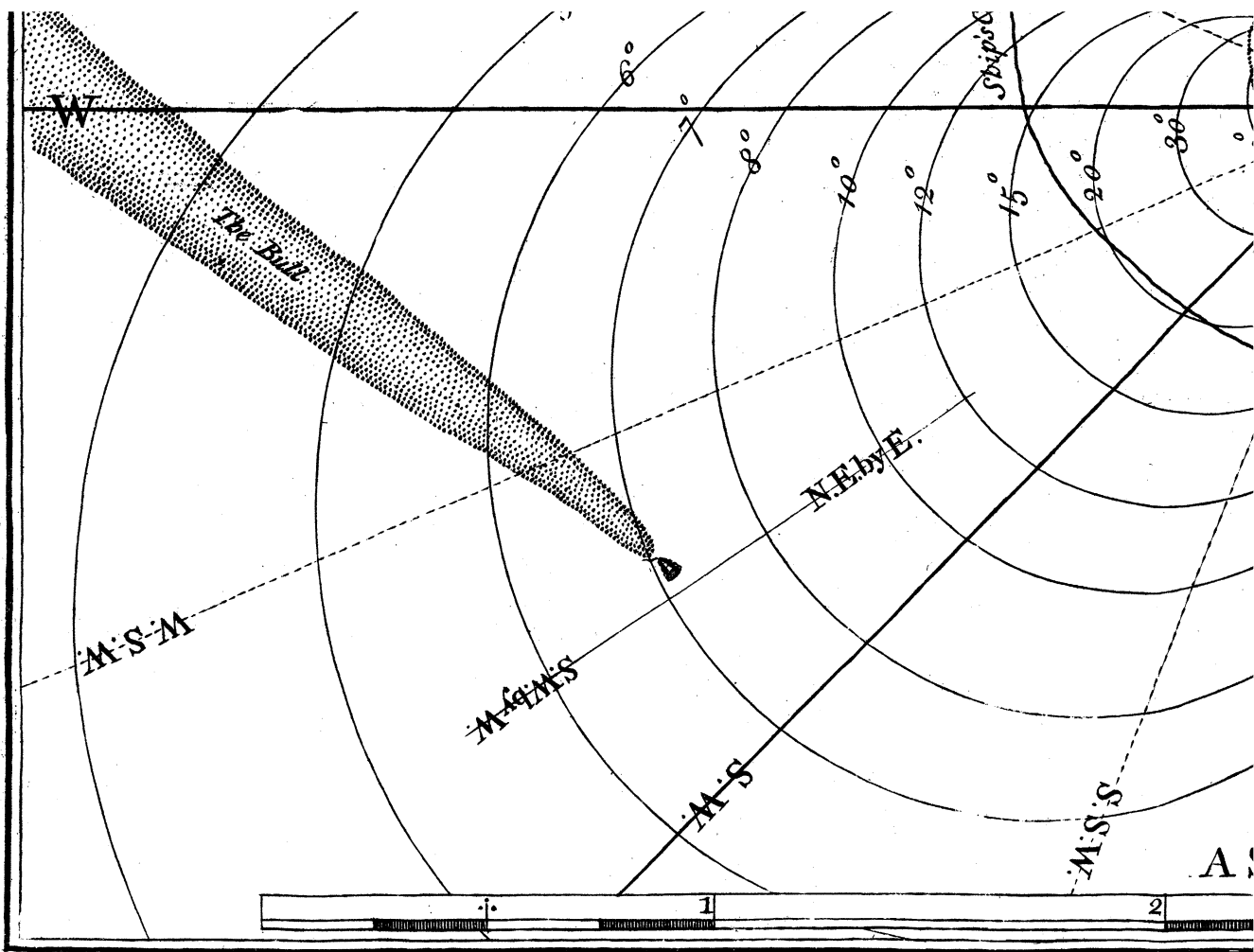I   the

Fig. 1.

C

D F A G H

120

B

Fig. 2.

M. N.

0° 30'

1°

1° 30'

M. N. W.

2°

2° 30'

3°

4°

5°

6°

K

I

M. N. N.
Trinity Sand

4°

5°

0° 30'
0° 30'
0° 30'
4°

Ship's Source

E

40

80

120

60

90

80

30

Magnetic.

N.N.E.

High Water mark

Kilnsea Com.

N.E.

D

Ship's Course

Low Water mark

Low Water mark

Old Spurn

Spurn Point

B

A. The place of the present Light

4° 5° 6° 7° 8° 10° 12° 15° 20° 30°

Course

E.N.E.

5°

4°

E

E

W

The Bull

6°

7°

8°

10°

12°

15°

20°

30°

Shipod

N.E. by E.

N.E. by N.

S.W. by W.

W. S. W.

S. W.

S. S. W.

A S

1

2

B

Spurn Point

C

Stone Banks

W. by N.

VII F

Crofs ridges & broken ground

VIII E

G

N. by W.

S

S.S.E.

S.E.

A Scale of five Miles.

2   3   4

E

VIII. F.
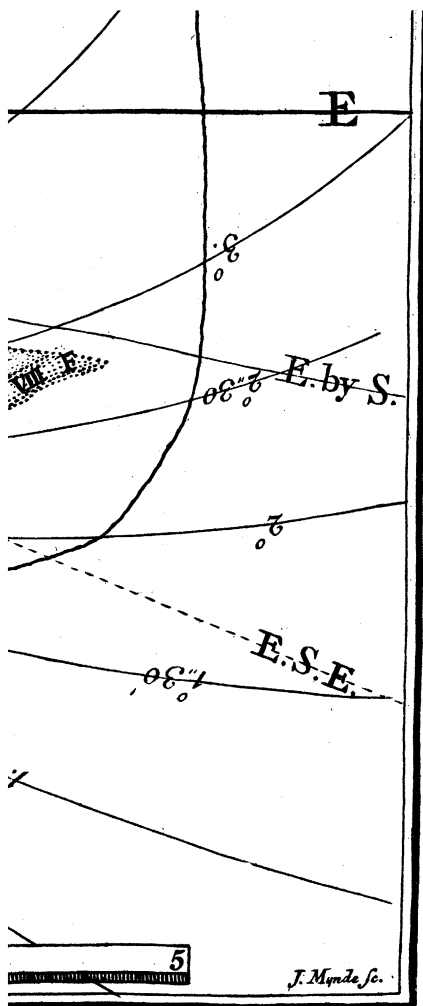
E. by S.

E.S.E.

5

J. Mynde sc.
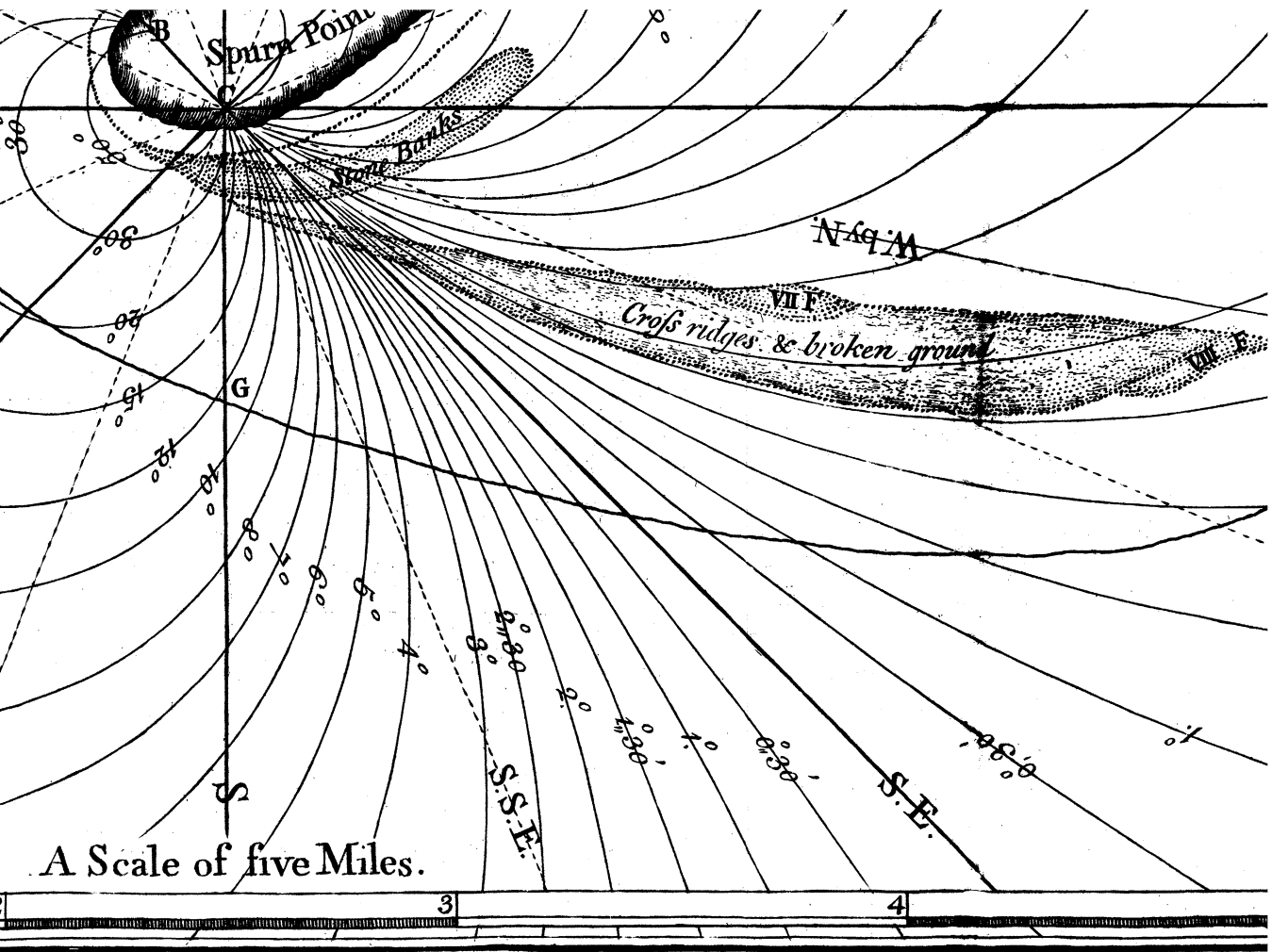
the intended fegments ; but, if the angle, corre-
fponding to the intended fegment, is to be neither
greater or lefs than 90°, the point A, which bifects
the line B C, will be the centre of the intended
fegment.

Thus, if I would have the angle in the intended
fegment, to confift of 120°, I conftitute upon
B C, at the point C, the angle D C B equal to
30°, the difference between 120° and 90°, and on
the oppofite fide of C B from its correfponding
fegment, becaufe it exceeds 90°. Then with the
centre D, and radius D C, I defcribe the fegment
C 120 B, in every part of which the two points
C and B will fubtend an angle of 120°.

In like manner, if I would have the angle in the
intended fegment to confift of 80°, I conftitute upon
B C, at the point C, the angle B C G equal to 10°,
the difference between 80° and 90°, and becaufe the
intended angle is now lefs than 90°, I place G on
the fame fide of B C with its correfponding feg-
ment. Then with the centre G, and radius G C,
I defcribe the fegment C 80 B, in every part of
which the two points C and B will fubtend an angle
of 80°.

Laftly, if I would have the angle, in the intended
fegment, to confift of 90°, with the centre A and
radius A C, I defcribe the femicircle C 90 B, in
every part of which the two points C and B will
fubtend an angle of 90° *.

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* If we want a fegment, the angle in which is to be the half
of one we have got before, the centre is already found to our
hands ; for it will be the point of interfection of the former
fegment with the perpendicular line D E. Thus the point,

The demonftration of all this is fo well known to every geometrician, that it needs not to be inferted ; but it may not perhaps be amifs to exemplify what I mean, by applying to it a particular cafe.

The prefent inconvenient and indeed dangerous fituation of the two lights at the mouth of the Humber, commonly called the Spurn Lights, muft probably foon make it neceffary to remove them ; for the ground, upon which they formerly ftood, is now fo far wafhed away, as not to leave fufficient room to erect them at a proper diftance from each other ; and frefh ground being grown up to the fouthward, fo as to make the point above a mile di- ftant from them, fhips are frequently thereby liable to be deceived. In cafe therefore thefe lights fhould, at any time hereafter, be removed nearer to the point, I think the foregoing principle might be very con- veniently and advantageoufly applied, fo as to enable entire ftrangers to enter the Humber, with the greateft

where the fegment, correfponding to $120°$, interfects the line D E, will be the centre of the fegment correfponding to $60°$: the point, where the fegment correfponding to $80°$, interfects the line D E, will be the centre of the fegment correfponding to $40°$, &c.

There is another method of finding the centres of thefe feg- ments, which, if great exactnefs is required, will perhaps be found preferable, in practice, to the former, efpecially in the larger fegments. The method, I mean, is to fet them off by a fcale, in which cafe the diftance of the feveral centres, from the point A, muft be refpectively proportional to the co-tangents of the angles in their correfponding fegments, or, what is the fame thing, refpectively proportional to the tangents of the angles D C A, F C A, G C A, H C A, &c. conftituted upon the line A C, at the point C ; for, making A C the radius, the lines D A, F A, G A, H A, &c. are the refpective tangents of thofe feveral angles.

fecurity,

fecurity, even in the darkeſt nights, provided only they could fee the lights: and this is the more material, as, in dangerous weather, it is frequently the beſt and moſt fecure retreat between the Thames and the Tyne ; but the difficulty of entering it, for want of proper helps, oftentimes obliges ſhips to keep the fea, not without great difficulty and hazard, when they might lie here in perfect fafety.

I have endeavoured to give ſome idea of the application, I propofe, in the annexed chart, [Fig. 2.] which is meant only as a rough draught of the general defign, and not as an accurate delineation of what muſt depend upon a future furvey, whenever the new lights ſhall be erected.

The ſtrong black line is intended to reprefent the high water mark : the ſpace included within this, and which is infcribed Spurn Point, as well as that infcribed Kilnfea Common, is ground, which the fea never overflows; but all the ſpace between thefe, where the fingle line only is continued, is frequently waſhed over by the fea, the land being there reduced, at the time of high water, to a meer bank of only a few yards wide.

The dotted line furrounding the ſtrong black line reprefents the low water mark ; and the other ſpaces included within dotted lines, and infcribed Trinity fand, the bull, crofs ridges and broken ground, and ſtone banks, are ſuppofed to reprefent four fands, the two former lying within, and the two latter without the mouth of the Humber.

Through the two points B and C, reprefenting the ſuppofed new lights, are defcribed, according to the rule above laid down, two feries of fegments, one

feries to the fouth weft, and the other to the north
eaft of them; but, the north eaftern feries lying
towards the land fide, a few of the inner fegments
might be omitted as being unneceffary.

From the point C are drawn lines in the direction
of fixteen of the points of the compafs; every other
point being omitted in order to make the chart more
diftinct; for no greater nicety is in general wanted,
than what thefe will afford; near the extremities
however of two of the fands, where it may perhaps
be more neceffary, I have added a fhort line to mark
the intermediate point.

I fhall now endeavour to illuftrate the whole by
an imaginary fhip's courfe. Let us fuppofe then a
fhip to enter the fpace, reprefented upon the chart,
from the north, and finding the light C to bear nearly
fouth weft, and the two lights to fubtend an angle
of about 5°, fhe will know herfelf to be fomewhere
about the place D, a little lefs eafternly, than the
point of the fand denominated crofs ridges and broken
ground, and at the diftance of about two miles and
a quarter from it, as may appear by the fcale; fhe
muft therefore fteer fomewhat to the eaftward of the
fouth, and, having run about three quarters of a mile,
fhe will find the light C to bear weft fouth weft, and
the two lights to fubtend an angle of exactly 5°, being
now in the interfection of the courfe, delineated upon
the chart, with the fegment fo marked; but not be-
ing yet far enough to the eaftward, fhe muft ftill con-
tinue the fame direction, till, having run near three
quarters of a mile farther, fhe will find the two lights
fubtending an angle of no more than 4°, and the light
C lying a little to the fouthward of the weft. From
hence

hence she may know herself to be at the place E, somewhat more to the eastward, than the sand she wants to avoid, and at the distance of almost a mile from it. She may now therefore safely change her course, and stand due south, till having brought the light C to bear west by north she is right over-against the extremity of the sand, at the distance of about three or four hundred yards. If she still keeps the same course about three hundred yards farther, the lights will then subtend an angle of only 2° 30', when she will be so far to the south, as to be clear of all danger, though she should from thenceforward steer due west; but, for greater security, we will suppose her still continuing to stretch away a little more to the southward, by steering south west for about a quarter of a mile farther. From hence then she may keep near due west, with which course the two lights, in a run of a mile and a quarter, or a little more, will both be brought to bear north west, and be seen together. In a run of about half a mile more, they will be again open on the contrary side from the former to about 3°, from which they will open to 10°, in a passage of near half a mile farther, when the ship will be arrived at the place G, the light C bearing then due north. The change of the angle being now very great in a very small distance, it will be very easy to keep the ship with great exactness to whatever part of the channel we please. Let us however suppose, that we mean to keep her in the course laid down in the chart; from G then we must steer north west a little west, and in a run of about a mile, we shall first increase the angle of the two lights, till it is somewhat more than 20°, the light C then bear-

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ing north eaſt, and afterwards diminiſh it again to about 13°, when the light C will bear due eaſt. Having now got far enough within the Spurn, we may run due north about a mile and half, firſt bringing the two lights to bear in one line due ſouth eaſt, and then opening them again, on the oppoſite ſide, to an angle of about 1° 30′ or 2°, according to our diſtance within the Spurn * ; but here, knowing perfectly well where we are, we can come to an anchor in a ſafe road, at ſuch a diſtance from the Trinity ſand as we pleaſe, which diſtance may be regulated without difficulty by the ſoundings, as at the place I, where the anchorage is very good in five fathom water.

* In the caſe of a ſhip's being placed nearly in the line of the two lights, the method of finding our diſtance from them by their bearing, and the angle they ſubtend, will fail us ; but, as it may ſometimes be convenient to have the means of forming a tolerable gueſs about our exact ſituation in this caſe, as well as others, I muſt obſerve that a pretty good judgment may be made concerning it, by running a little way in a direction nearly at right angles to the line of the lights ; for by this means, the diſtance run being known, it will appear, from the change of the angle, under which they are ſeen, how far they are from us. Thus, for example, if from the place I, a veſſel ſhould run about a quarter of a mile along the dotted line I K, and find, that this made a change in the angle ſubtended by the two lights of a very little more than a degree, ſhe would know herſelf to be almoſt two miles from the light C, as would appear by meaſuring the diſtance I C upon the ſcale.

Fig. 1.

Fig. 2.

A Scale of five Miles.

J. Mynde Sc.